Roll No. .... **Total No. of Pages : 7**

1(CCE.M)3

Statistics–I

(23)

Time : Three Hours] [Maximum Marks : 300

INSTRUCTIONS

(i) Answers must be written in English.

(ii) The number of marks carried by each question is indicated at the end of the question.

(iii) The answer to each question or part thereof should begin on a fresh page.

(iv) Your answer should be precise and coherent.

(v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.

(vi) Candidates should attempt any **five** questions choosing at most **two** from each Section.

(vii) If you encounter any typographical error, please read it as it appears in the text-book.

(viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.

(ix) No continuation sheets shall be provided to any candidate under any circumstances.

(x) Candidates shall put a cross (x) on blank pages of Answer Script.

10. (a) Define correlation coefficient and correlation ratio. State the properties of correlation coefficient and prove any one of them. 15

(b) The Joint density function of x and y is given by

$$f(x, y) = \begin{cases} x + y, & 0 < x < 1, \ 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

obtain the regression curve of y on x. 15

(c) If x and y are standard normal variates with coefficient of correlation ρ, show that regression of y on x is linear. 15

(d) Define bivariate normal distribution. If (x, y) has a bivariate normal distribution, find the marginal density function $f_x(x)$ of x. 15

11. (a) Explain the concepts of multiple and partial correlation coefficients. Show that the multiple correlation coefficient $R_{1.23}$ is, in usual notations given by $R^2_{1.23} = 1 - \frac{w}{w_{11}}$. 20

(b) The simple correlation coefficient between temperature (x_1) corn yield (x_2) and rainfall (x_3) are $r_{12} = 0.59$, $r_{13} = 0.46$ and $r_{23} = 0.77$. Calculate the partial correlation coefficients $r_{12.3}$ and $r_{23.1}$ and also calculate $R_{1.23}$. 20

(c) With usual notation prove that :

$$R^2_{1.23} = b_{12.3}\ r_{12} \frac{\sigma_2}{\sigma_1} + b_{13.2}\ r_{13} \frac{\sigma_3}{\sigma_1}.$$ 20

(c) If the joint probability density of x and y is given by :

$$f(x, y) = \begin{cases} x + y & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

find the corresponding joint distribution function. 15

(d) Two random variables x and y have the following joint density function :

$$f(x, y) = \begin{cases} 2 - x - y & ; 0 \le x, y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

find Var (x), Var (y) and covariance between x and y. 15

3. (a) Define convergence in probability and convergence in distribution with an example each. 15

(b) State and prove Chebyshev's theorem. 15

(c) State and prove Borel-Cantelli Lemma. 15

(d) For the following sequence of Independent Random Variables, does weak law of large number hold :

$$P\left\{x_k = \pm 2^k\right\} = \frac{1}{2}.$$ 15

4. (a) Define Moment Generating Function of a random variable. If M (t) is the m.g.f. of a random variable x about origin, show that the moment μ'_v is given by :

$$\mu'_v = \left[\frac{d^v\ M(t)}{dt^v}\right] t = 0.$$ 15

(b) State and prove strong law of large numbers. 15

(c) Define the characteristic function of a random variable. Show that the characteristic function of the sum of two independent variables is equal to the product of their characteristic functions. 15

(d) Define central limit theorem and state its importance. 15

SECTION–II

5. (a) Discuss the terms :

   (i) Estimate

   (ii) Consistent estimate

   (iii) Unbiased estimate

   of parameter and show that sample mean is both consistent and unbiased estimate of the population Mean. 15

   (b) Define MVU estimator. Show that an MVU estimator is unique. 15

   (c) How is Cramer-Rao inequality useful in obtaining MVUE ? Derive this inequality. 15

   (d) State and prove Rao-Blackwell theorem. 15

6. (a) State and explain the principle of Maximum Likelihood for estimation of population parameter. Discuss its properties. 15

   (b) Describe the method of moments for estimating the parameters. What are the properties of the estimates obtained by this method ? 15

   (c) Discuss the concept of Interval estimation. Obtain the minimum confidence interval for the variance for a random sample of size 'n' from a normal population with unknown mean. 15

   (d) Discuss the general method of construction of likelihood ratio test. 15

7. (a) What are the advantages and drawbacks of non-parametric methods over parametric methods ? 15

   (b) Develop the Mann-Whitney test. Obtain the Mean and Variance of Statistic T. 15

   (c) Describe the Median test for the two sample location problem. 15

   (d) What is a sequential test ? Describe Wald's Sequential Probability Ratio Test. 15

8. (a) What is meant by a statistical hypothesis ? Explain the concepts of type-I and type-II errors. 15

   (b) State and prove Neyman-Pearson Lemma for testing a simple hypothesis against a simple alternative. 15

   (c) Let 'p' denote the probability of getting a head, when a coin is tossed once. Suppose that the hypothesis $H_o : p = 0.5$ is rejected in favour of $H_1 : p = 0.6$ if 10 trials result in 7 or more heads. Calculate the probabilities of type-I and type-II errors. 15

   (d) Obtain the most powerful test for testing the mean, $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$ $(\mu_1 > \mu_0)$ when $\sigma^2 = 1$ in normal population. 15

SECTION–III

9. (a) Define Hotelling's T^2-statistic. What are its Merits and Limitations in Multivariate data analysis ? 20

   (b) What is Mahalanobis distance ? What are the applications of Mahalanobis D^2 distribution ? 20

   (c) Describe Fisher's discriminant analysis. Explain its importance. 20

(xi) No blank page be left in between answer to various questions.

(xii) No programmable Calculator is allowed.

(xiii) No stencil (with different markings) is allowed.

SECTION–I

1. (a) Four candidates have applied for a Teacher's Job. If A is twice as likely to be selected as B, and B and C have equal chance of getting selected, while C is twice as likely to be selected as D, what are the probabilities that :

(i) C gets selected

(ii) A is not selected ? 15

(b) Prove by induction that $p\,(A_1 \cup A_2 \cup \ldots \cup A_n) \le \sum_{i=1}^{N} p\,(A_i)$

for any finite events A_1, A_2, ..., and A_n. 15

(c) Show that $2^u - u - 1$ conditions must be satisfied for k events to be independent. 15

(d) State and prove Bayes' theorem. 15

2. (a) Define Joint and Marginal density function. Find the Joint Marginal density of x_1 and x_3 and the Marginal density of x_1 for the following trivariate density function :

$$f\,(x_1, x_2, x_3) = \begin{cases} (x_1 + x_2)\,\bar{e}^{x_3}, & \text{for} \quad 0 < x_1 < 1,\ 0 < x_2 < 1,\ x_3 > 0. \\ 0, & \text{elsewhere} \end{cases}$$

15

(b) Let x be a Random Variable such that $E \mid x \mid < \infty$. Show that $E \mid x - c \mid$ is minimised if we choose 'c' equal to the median of the distribution of x. 15

12. (a) State Gauss-Markoff Lemma. 5

(b) For a one-way classified fixed effect model $x_{ij} = \mu + \alpha_i + \in_{ij}$ $(i = l, \ldots\ldots, k, j = 1, 2, \ldots\ldots., n_k)$ obtain the estimators of parameters μ and α_i's and the expectations of the various sum of squares. 20

(c) Outline the various steps in carrying out the ANOVA of a twoway classified data with one observation per cell. 15

(d) Discuss the method of fitting an orthogonal polynomials. What are the advantages of using orthogonal polynomials for fitting curvilinear relations ? 20