

- (c) Explain how do you fit orthogonal polynomials and mention the importance of such curves fitting. 20
12. (a) Describe the principle Component Analysis. 20
- (b) Explain the Method of Discriminant Analysis when two Normal populations with K-variables. 20
- (c) What is Mahalanobis distance and write about Mahalanobis- $D^2$  distribution. 20

Roll No. ....

Total No. of Pages : 6

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**Statistics**

**(23)**

**Paper—I**

Time : Three Hours]

[Maximum Marks : 300

- Note :—**
- (i) Answers must be written in English.
  - (ii) Number of marks carried by each question are indicated at the end of the question.
  - (iii) Part/Parts of the same question must be answered together and should not be interposed between answers to other questions.
  - (iv) The answer to each question or Part thereof should begin on a fresh page.
  - (v) Your answers should be precise and coherent.
  - (vi) Attempt **five** questions in all, choosing at most **two** questions from each Section. Question No.1 is compulsory.
  - (vii) If you encounter any typographical error, please read it as it appears in the text-book.

**SECTION-I**

1. (a) (i) Define the classical probability and justify the limits of it. 20
- (ii) State and prove Addition theorem of probability for two events. Comment on the property when the events are disjoint. 20
- (b) Write about the method of moments. Also state the merits and limitations of this method. 20
- (c) Write about the Gauss Markoff theorem. State its importance in Analysis of variance. 20

2. (a) State and prove Chebychev's Inequality. 30
- (b) A class consists of equal number of male and female students. An experiment of selecting a male student is being conducted. Let 'P' be the proportion of selecting a male student among the total students. How many times the experiment should be conducted in order that the probability will be at least 0.9 such that  $0.4 \leq P \leq 0.6$ , while using Chebychev's Inequality? 30
3. (a) Define Sample Space, and Probability Space with suitable Examples. 15
- (b) Define Conditional Probability and show that it satisfies Additive Law. 15
- (c) State and Prove Bayes' Theorem. 15
- (d) An egg distribution center is getting the products from 3 Poultrys A,B,C with the ratio 1:2:3. The chances of getting rotten egg from the poultrys A,B,C respectively are 0.1, 0.05, 0.025. During a sample examination, an egg from the stock point was picked up randomly, and it was found rotten. What are the chances that egg has come from poultry B&C? 15
4. (a) Let 'X' be a random variable denoting the number of organs which are functioning properly. The following is a probability Distribution for X.
- |       |     |     |     |      |      |
|-------|-----|-----|-----|------|------|
| X :   | 0   | 1   | 2   | 3    | 4    |
| P(X): | 0.1 | 0.2 | 0.3 | 0.15 | 0.25 |
- Find the probability of
- (i) At least one of the organs work properly
- (ii) At least 3 of the organs work properly
- (iii) At least 2 and at most 4 organs work properly. 15
- (b) Probability of speaking truth by A and B are 0.35 and 0.40

respectively. If both of them comment on a statement, what are the chances that

- (i) Both will contradict each other
- (ii) Both will speak the same
- (iii) None of them speak truth
- (iv) Both of them will speak truth? 20
- (c) Let X be a random variable such that  $P(X < 0) = P(X = 0) = P(X > 0)$ ;  
 $P(X = -1) = P(X = -2) = P(X = -3)$ ;  
 $P(X = +1) = P(X = +2) = P(X = +3)$ ; Then write a
- (i) Probability distribution of X
- (ii) Probability distribution of Y, where  $Y = 2X^2 + 3X + 4$ . 10
- (d) Find the value of K, Mean, and Variance of X for a p.d.f  $f(x) = K(2-x).x$ ;  $0 \leq x \leq 2$ ;  $f(x) = 0$  otherwise. 15

### SECTION-II

5. (a) Define Consistency, Unbiasedness, and Sufficiency; give suitable examples. 15
- (b) Let 'X' be a Bernoulli variable having 0,1 with probability  $(1-\theta)$  and  $\theta$  respectively, then find the unbiased estimator of  $T(T-1)$  where  $T = \sum_{i=1}^n X_i$  10
- (c) If  $T_1$  is an MVUE of  $\gamma(\theta)$ ;  $\theta \in \Theta$  and  $T_2$  is any other unbiased Estimator of  $\gamma(\theta)$  with efficiency  $=e$ ; then show that the correlation coefficient between  $T_1, T_2 = \sqrt{e}$  15
- (d) Find the Best Critical Region for sample Mean, when a sample of 'n' independent observations is drawn from a Normal

population with Mean= $\mu$  and Variance=1 under  $H_0: \mu = \mu_0$  against  $H_1: \mu = \mu_1$ .

20

6. (a) State and Prove Cramer-Rao Inequality. State its importance.

20

(b) Find the Minimum Variance bound estimator of sample mean when a sample of 'n' independent observations is drawn from a Normal Population with Mean =  $\mu$  and Variance =  $\sigma^2$  (known)

20

(c) State Neyman's Factorization theorem, using it find sufficient estimator of  $\lambda$  when a sample  $(X_1, X_2, X_3, \dots, X_n)$  is drawn from the Poisson population with P.m.f.  $P(x) = e^{-\lambda} \lambda^x / x!$ ;  $\lambda > 0$ ;  $x = 0, 1, 2, \dots$ ;  $P(x) = 0$ ; otherwise.

20

7. (a) Define Best Critical Region; Size of Critical Region and Power of the Test.

15

(b) Show that every most powerful critical region is necessarily unbiased.

15

(c) Define parameters space, Likelihood Ratio Test and give the properties LR test.

15

(d) Describe the test procedure in Large sample case for difference between two population standard deviations.

15

8. (a) Write the Non-Parametric Test Procedure using Mann-Whitney-Wilcoxon test.

15

(b) Let 'X' be a random variable, with p.m.f.

$$P(x; \theta) = \theta^x (1-\theta)^{1-x}; x=0, 1; 0 < \theta < 1;$$

$P(x; \theta) = 0$ ; elsewhere; for Testing  $H_0: \theta = \theta_0$  against  $H_1: \theta = \theta_1$ .

construct SPRT and obtain OC and ASN functions. 30

(c) Find the size of critical region ( $\alpha$ ); power of the test ( $1-\beta$ ) for a critical region  $W = \{1 \leq x \leq 1.5\}$  under  $H_0: \lambda = 1$  against  $H_1: \lambda = 2$  by Mean of single observation belong to a population having p.d.f.  $f(x, \lambda) = 1/\lambda$ ;  $0 \leq x \leq \lambda$ ;

$f(x, \lambda) = 0$ ; elsewhere. 15

### SECTION-III

9. (a) Derive the Normal equations for power curve and Exponential curves using the principles of least squares. 20

(b) Define Bivariate Normal density function and explain the Bivariate Normal distribution including Merits and Limitations. 20

(c) Compare the concepts of simple Regression and simple Correlation. 20

10. (a) Write the importance of Hotelling's  $T^2$  statistic in inference and explain how inference can be done with Mean Vector. 20

(b) Write the Bonferroni Method of Multiple Comparisons. 20

(c) Write about One Multivariate Analysis of variance (MANOVA). 20

11. (a) Write about the Classical linear regression Model in Multivariate Analysis of Variance. 20

(b) Describe Multivariate Multiple Regression. 20