Total No. of Printed Pages : 7 **Roll No. ....**

1[CCE.M]1

Mathematics–II

(15)

Time : Three Hours Maximum Marks : 300

INSTRUCTIONS

(i) Answers must be written in English.

(ii) The number of marks carried by each question is indicated at the end of the question.

(iii) The answer to each question or part thereof should begin on a fresh page.

(iv) Your answers should be precise and coherent.

(v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.

(vi) Candidates should attempt any **five** questions.

(vii) If you encounter any typographical error, please read it as it appears in the text book.

(viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.

(ix) No continuation sheets shall be provided to any candidate under any circumstances.

11. (a) Derive equation of continuity in vector form. 30

(b) A sphere of radius a is surrounded by infinite liquid of density ρ, the pressure at infinity being π. The sphere is suddenly annihilated. Show that the pressure at a distance r from the centre immediately falls to $\pi\left(1-\frac{a}{r}\right)$. 30

12. (a) A velocity field is given by $\vec{q} = -xi + (y+t)j$. Find the stream function and the stream lines for this field at t = 2. 20

(b) A two-dimensional flow field is given by $\psi = xy$.

(i) Show that the flow is irrotational

(ii) Find the velocity potential

(iii) Verify that ψ and ϕ satisfy the Laplace equation. 20

(c) Discuss source and sinks in two-dimension. 20

13. (a) Evaluate $\int_{-3}^{3} x^4 dx$ by taking seven equidistant ordinates. Compare it with the exact value. 30

(b) Given $\frac{dy}{dx} = \frac{1}{x+y}$, y(0) = 2. If y(0.2) = 2.09, y(0.4) = 2.17 and y(0.6) = 2.24 find y(0.8) using Milne's method. 30

14. (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 51 and x = 85 from the following data :

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

30

using suitable interpolation formulae.

(b) Show that an infinite series $\sum_{n=1}^{\infty} \frac{a_n}{10^n}$ where each a_n is an integer, $0 \le a_n \le 9$ converges. 20

(c) Let $\langle M_1, P_1 \rangle$ be a compact matric space f is a continuous function from M_1 into a metric space $\langle M_2, P_2 \rangle$. Then f is uniformly continuous on M_1. 20

4. (a) Find the maxima and minima of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$ 20

(b) If a double integral, $I = \iint_R f\, dx\, dy$ exists over a rectangle R = (a, b; c, d), and if $\int_c^d f\, dy$ also exists, for each fixed x in [a, b], then prove that $\int_a^b dx \int_c^d f\, dy$ exists and is equal to the double integral I. 20

(c) (i) If f is continuous on [a, b] and α has a continuous derivative on [a, b] then prove that $\int_a^b f\, d\alpha = \int_a^b f\, \alpha'\, dx$.

(ii) Show that the integral $\int_0^{\pi/2} \log \sin x\, dx$ is convergent and hence evaluate it. 20

5. (a) If f(z) is analytic in a simply connected domain D bounded by rectifiable Jordan Curve C and is continuous on 'C' then

$$f(z) = \frac{1}{2\pi i} \int_c \frac{f(\xi)}{\xi - Z}\, d\xi$$

where Z is any point in D. 20

(b) Find the domain of Convergence of the power series $\left(\frac{iz-1}{2+i}\right)^n$. 20

(c) Let f = u + iv be holomorphic on an open set U. Then both the partials of both u and v exist on U and for any $z = x + iy \in U$; $u_x = v_y$ and $v_x = -u_y$. 20

6. (a) Show that :

$$f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^2} & \text{where } z \neq 0 \\ 0 & \text{where } z = 0 \end{cases}$$

is not differentiable at z = 0. 20

(b) Prove that the function $u = e^x (x \cos y - y \sin y)$ satisfy Laplace's equation and find the corresponding analytic function f(z) = u + iv. 20

(c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}$. 20

7. (a) Use Charpits method to solve the following partial differential equations :

(i) $(p^2 + q^2)\, y = qz$

(ii) $z^2 = pqxy$. 20

(b) (i) Solve $(x^2 - yz)\, p + (y^2 - zx)\, q = z^2 - xy$.

(ii) Find the general solution of the partial differential equation $px\,(x + y) = qy\,(x + y) + (x - y)\,(2x + 2y + z)$. 20

(c) (i) Solve $D^3 - 6D^2 D' + 11 D D'^2 - 6D'^3)z = 0$

(ii) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$. 20

8. (a) Solve the equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$. 20

(b) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to Canonical form. 20

(c) Find a complete integral of $pq = x^m y^n z^{2l}$. 20

SECTION–B

9. (a) A solid body, if density ρ, is in the shape of the solid formed by the revolution of the Cardiod $r = a(1 + \cos\theta)$ about the initial line; show that its moment of inertia about a straight line through the pole perpendicular to the initial line is $\left(\frac{352}{105}\right)\pi\rho a^5$. 30

(b) A particle of mass m moves in a conservative force field. Find the Lagrangian function and the equation of motion in cylindrical co-ordinates (ρ, φ, z). 30

10. (a) A mass M hangs from a fixed point at the end of a very long string whose length is a, to M is suspended a mass m by a string whose length ℓ is small compared with a, prove that the time of a small oscillation of m is $\sqrt{\left(\frac{M}{M+m} \cdot \frac{\ell}{g}\right)}$. 30

(b) A cannon of mass M, resting on a rough horizontal plane of coefficient of friction μ, is fired with such a charge that the relative velocity of the ball and cannon at the moment when it leaves the cannon is u. Show that the cannon will recoil a distance

$$\left(\frac{mu}{M+m}\right)^2 \cdot \frac{I}{2\mu g}$$

along the plane, m being the mass of the ball. 30

(x) Candidates shall put a cross (X) on blank pages of Answer Script.

(xi) No blank page be left in between answer to various questions.

SECTION–A

1. (a) If H is a nonempty finite subset of a group G and H is closed under multiplication, then H is a subgroup of G. 20

(b) If H is a subgroup of G and N is a normal subgroup of G then show that $H \cap N$ is a normal subgroup of H. 20

(c) HK is a subgroup of G if and only if HK = KH. 20

2. (a) Given two polynomials f(x) and $g(x) \neq 0$ in F[x], then there exist two polynomials t(x) and r(x) in F[x] such that f(x) = t(x) g(x) + r(x) where r(x) = 0 or deg r(x) < deg g(x). 20

(b) Let $P \in Z$ be a prime. Suppose that $f(x) = a_n x^n + a_{n-1} x^n + ... + a_0$ is in Z[x] and $a_n \not\equiv 0 \pmod p$, but $a_i \equiv 0 \pmod p$ for all i < n, with $a_0 \not\equiv 0 \pmod{p^2}$. Then f(x) is irreducible over Q. 20

(c) Show that every finite integral domain is a field. 20

3. (a) Let ℓ_2 be the set of all real sequence $\{x_n\}$, for which the series $\sum_{n=1}^{\infty} x_n^2$ converges. Show that the function d defined by

$$d(\{x_n\},\{y_n\}) = \left[\sum_{n=1}^{\infty}(x_n - y_n)^2\right]^{1/2}, \{x_n\}, \{y_n\} \in \ell_2 \text{ is a metric}$$

space. 20

(b) Find the root of the equation 3x – cos x – 1 = 0 by using Newton-Raphson method correct to 4 decimal places. 30

15. (a) State and Prove addition theory of probability for two events. 20

(b) Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys and 1 girl and 3 boys. One child is selected at random from each group. Show that the chance that the three selected consist of 1 girl and 2 boys is 13/32. 20

(c) The contents of Urns I, II and III are as follows :

1 white, 2 black and 3 red balls

2 white, 1 black and 1 red balls, and

4 white, 5 black and 3 red balls

One urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urns I, II or III ? 20

16. (a) Obtain necessary and sufficient conditions for the optimum solution of the following Non-linear programming problem :

Min z = $f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$

Subject to constraint

$x_1 + x_2 = 7, x_1, x_2 \geq 0.$ 30

(b) Two companies A and B are competing for the same product; their different strategies, are given in the following pay off matrix :

Company A

$$\text{Company B}\begin{bmatrix} 2 & -2 & 3 \\ 3 & 5 & -1 \end{bmatrix}$$

Use linear programming to determine the best strategies for both the players. 30