Roll No. .... **Total No. of Pages : 7**

1(CCE.M)3

Mathematics–II

(15)

Time : Three Hours] [Maximum Marks : 300

INSTRUCTIONS

(i) Answers must be written in English.

(ii) The number of marks carried by each question is indicated at the end of the question.

(iii) The answer to each question or part thereof should begin on a fresh page.

(iv) Your answer should be precise and coherent.

(v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.

(vi) Candidates should attempt any **five** questions.

(vii) If you encounter any typographical error, please read it as it appears in the text-book.

(viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.

(ix) No continuation sheets shall be provided to any candidate under any circumstances.

(x) Candidates shall put a cross (x) on blank pages of Answer Script.

(b) Calculate the Pearson's coefficient of skewness from the following table :

Wages (Rs.)	0-10	10-20	20-30	30-40	40-50
No. of workers	15	20	30	25	10

20

(c) The ranks of the same 15 students in two subjects A and B are given below. The two numbers within the brackets denote the rank of the same student A and B respectively. (1, 10), (2, 7), (3, 2), (4, 6), (5, 4), (6, 8), (7, 3), (8, 1), (9, 11), (10, 15), (11, 9), (12, 5), (13, 14), (14, 12), (15, 13). Calculate the rank correlation coefficient. 20

14. (a) State and prove the addition theorem of probability for any two events A and B. Rewrite the theorem when A and B are mutually exclusive. 20

(b) In a class of 75 students, 15 were considered to be very intelligent, 45 as medium and the rest below average. The probability that a very intelligent student fails in a viva-voce examination is 0.005 ; the medium student failing has a probability 0.05 ; and the corresponding probability for a below average student is 0.15. If a student is known to have passed the viva-voce examination, what is the probability that he is below average ? 20

(c) Define Binomial distribution. What is the probability of guessing correctly at least six of the ten answers in a TRUE-FALSE objective test ? 20

15. (a) Using the simplex method, find non-negative values of x_1, x_2 and x_3 which maximize $Z = x_1 + 9x_2 + x_3$ subject to the constraints $x_1 + 2x_2 + 3x_3 \le 9$ and $3x_1 + 2x_2 + 2x_3 \le 15$. 20

4. (a) If P* is a refinement of P, then prove that :

(i) $L(P, f, \alpha) \le L(P^*, f, \alpha)$ and

(ii) $\cup(P^*, f, \alpha) \le \cup(P, f, \alpha)$. 20

(b) Discuss the convergence of the improper integral :

$$\int_a^b \frac{1}{(x-a)^n}\,dx.$$ 20

(c) Show that $f(x, y) = x^4 + y^2 + x^2y$ has a minimum at (0, 0). 20

5. (a) Show that

$$f(z) = \begin{cases} \dfrac{xy^2(x+iy)}{x^2+y^4} & \text{if } z \ne 0 \\ 0 & \text{if } z = 0 \end{cases}$$ is not differentiable at $z = 0$. 20

(b) Show that $u = \log\sqrt{x^2+y^2}$ is harmonic and determine its conjugate. Also find the Correponding analytic function. 20

(c) If f (z) is analytic prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f^1(z)|^2.$$ 20

6. (a) Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$ where C is the circle $|z| = 3$. 20

(b) Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's series :

(i) About the point $z = 0$

(ii) About the point z = 1. Determine the region of convergence in each case. 20

(c) Using contour integration evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$. 20

7. (a) Form a partial differential equation by eliminating a, b, c from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ 20

(b) Form a partial differental equation by eliminating the arbitrary function ϕ from $\phi(x + y + z, x^2 + y^2 - z^2) = 0$. What is the order of this partial differential equation. 20

(c) Solve :

$$x(y^2 + z)P - y(x^2 + z)q = z(x^2 - y^2).$$ 20

8. (a) Using Charpit's method solve the partial differential equation $ZPq = P + q$. 20

(b) Solve :

$$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial y\,\partial x} + 2\frac{\partial^2 z}{\partial y^2} = x + y.$$ 20

(c) Solve :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\,\partial y} - 6\frac{\partial^2 z}{\partial y^2} = \text{Cos}(2x + y).$$ 20

SECTION–B

9. (a) Write a note on holonomic and non-holonomic constraints with two examples of each type. 20

(b) State and prove D'Alembert's principle. 20

(c) Obtain the equation of motion of a simple pendulum by using Lagrangian method and hence deduce the formula for its time period for small amplitude oscillations. 20

10. (a) Derive the equation of Continuity in Carterian co-ordinates. 30

(b) Discuss sources and sinks in two-dimension. 30

11. (a) Find one root of $x^3 - 9x + 1 = 0$ by bisection method correct to three decimal places. 20

(b) Find the root of $x^3 - 8x - 4 = 0$ which lies between 3 and 4 by Newton-Raphson method correct to four decimal places. 20

(c) Find the interpolating polynomial f (x) satisfying f (0) = 0, f (2) = 4, f (4) = 56, f (6) = 204, f (8) = 496 and f (10) = 980. 20

12. (a) Find $\left(\frac{dy}{dx}\right)_{n=1}$ and $\left(\frac{d^2y}{dx^2}\right)_{x=3}$ from the following table :

x	0	2	4	6	8
y	7	13	43	145	367

20

(b) Evaluate $\int_0^6 3x^2dx$ by taking seven equidistant ordinates. 20

(c) Using Runge-Kutta method of fourth order find y (0.2) for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, y (0) = 1 by taking h = 0.2. 20

13. (a) Calculate the quartile deviation for the following fequency dirtribution :

x	60	62	64	66	68	70	72
frequency	12	16	18	20	15	13	9

20

(xi) No blank page be left in between answer to various questions.

(xii) No programmable Calculator is allowed.

(xiii) No stencil (with different markings) is allowed.

SECTION–A

1. (a) Prove that the relation isomorphism in the set of all groups is an equivalence relation. 20

(b) State and prove Cayley's theorem for finite groups. 20

(c) Let G be a finite group such that $P^m \mid o(G)$ and $P^{m+1} \nmid o(G)$, where P is a prime number and m is a positive integer. Then show that G has subgroups of order P, P^2, ...P^m. 20

2. (a) Show that :

$$Q(w) = \{a + bw \mid a, b \in Q, 1 + w + w^2 = 0, w^3 = 1\}$$

is a field under addition and multiplication of complex numbers. 20

(b) Show that every field in an Euclidean ring. 20

(c) If M is a finite extension of a field K and K is finite extension of a field F such that $F \subset K \subset M$ then show that M is a finite extension of F and $[M : F] = [M : K] [K : F]$. 20

3. (a) Let (X, d) be a metric space, and $d_1(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$

for all $x, y \in X$. Then show that d_1 is a metric on X. 20

(b) Define Cauchy sequence. Show that in any metric space X, every convergent sequence is a Cauchy sequence. 20

(c) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X. 20

(b) Define feasible solution, basic solution, non-degenerate solution and optimal solution in a transportation problem. 20

(c) Solve the following non-linear programming problem by using Kuhn-Tucker conditions :

$$\text{Maximize } Z = x_1^2 - x_1x_2 - 2x_2^2$$
$$\text{Subject to } 4x_1 + 2x_2 \le 24$$
$$5x_1 + 10x_2 \le 20$$
$$x_1, x_2 \ge 0.$$

20

16. (a) A tax consulting firm has 3 counters in its office to receive people who have problems concerning their income, wealth and sales taxes. On the average 48 persons arrive in an 8-hour day. Each tax adviser spends 15 minutes on an average on an arrival. If the arrivals are Poissonly distributed and service times are according to exponential distribution, find :

(i) The average number of customers in the system

(ii) Average number of customers waiting to be served

(iii) Average time a customer spends in the system. 30

(b) There are seven jobs, each of which has to go through the machines A and B in the order AB. Processing time in hours are given as :

Job	1	2	3	4	5	6	7
Machine A	3	12	15	6	10	11	9
Machine B	8	10	10	6	12	1	3

Determine a sequence of these jobs that will minimize the total elapsed time T. Also find T and idle time for machines A and B. 30