This question paper contains **15** printed pages]

Code No. : 15(II) *Roll No. ....*

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MATHEMATICS

Paper : II

Time Allowed : 3 hours] ***[Maximum Marks : 300***

Note : *(i) Answers must be written in* ***English.***

(ii) Number of marks carried by each question are indicated at the end of the question.

(iii) Part/Parts of the same question must be answered together and should not be interposed between answers to other questions.

(iv) The answer to each question or part thereof should begin on a fresh page.

(v) Your answers should be precise and coherent.

(vi) Attempt ***five*** *questions in all, selecting at least* ***two*** *questions from each of the Section A & B.*

P. T. O.

SECTION – A

1. (a) If a Group G has four elements, show that it must be abelian.

   (b) If G is a group and H is a subgroup of index 2 in G, prove that H is a normal subgroup of G.

   (c) A ring R is without zero divisors if and only if the Cancellation laws hold in R, i. e., R is without zero divisors $\Leftrightarrow$ Cancellation laws hold in R. 20, 20, 20

2. (a) (i) Every field is an integral domain.

   (ii) The characteristic of an integral domain is either 0 or a prime number.

   (b) Let R be a commutative ring with unit element. Let S be a maximal ideal of R is a prime ideal.

   (c) If R is a unique factorization domain, then the product of two primitive polynomials in $R[x]$ is again a primitive polynomial in $R[x]$. 20, 20, 20

3. (a) Let f be a real valued function on a metrix space X. Prove that f is continuous of X if and only if the following sets : $\{x : f(x) < c\}$ and $\{x : f(x) > c\}$ are open in X for every $C \in R$.

(b) If $(x, y, z) = \dfrac{(a^2x^2 + b^2y^2 + c^2z^2)}{x^2y^2z^2}$, where $ax^2 + by^2 + cz^2 = 1$, and a, b, c are positive, show that the minimum value of $f(x, y, z)$ is given by :

$$x^2 = \frac{u}{2a(u+a)}, y^2 = \frac{u}{2b(u+b)}, z^2 = \frac{u}{2c(u+c)},$$

Where u is the positive root of the equation :

$$u^3 - (bc + ca + ab)u - 2abc = 0$$

(c) Discuss the convergence of the integral :

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$$

20, 20, 20

4. (a) Show that the series $\sum \dfrac{x}{n^p + x^2n^q}$ converges uniformly over any finite interval $[a, b]$, for :

(i) $p > 1,\ q \geq 0$

(ii) $0 < p \leq 1,\ p + q > 2$

(b) (i) Construct a function $f(z)$ which has a real function $u(x, y) = e^{2x}(x \cos 2y - y \sin 2y)$ for its real part satisfying Laplace's equation.

(ii) Examine the nature of singularities of $\frac{z\, e^{iz}}{(z^2 + a^2)}$ and find the residues at the singularities.

(c) If a function f is continuous on [0, 1].

Show that :

$$\lim_{n \to \infty} \int_0^1 \frac{n\, f(x)}{1 + n^2 x^2}\, dx = \frac{\pi}{2} f(0)$$ 20, 20, 20

5. (a) Expand :

$$\frac{1}{(1 + z^2)(z + 2)}$$

When (i) $|<|z|<2$

(ii) $|z|>2$

(b) (i) Find the kind of singularity of the function :

$$f(z)=\left\{\sin\left(\frac{1}{z}\right)\right\}^{-1}$$

(ii) Show that the function $z|z|$ is not analytic anywhere.

(c) If $f(z)$ is an analytic function of z, in any domain, show that :

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^p=p^2|f(z)|^{p-2}|f'(z)|^2$$

20, 20, 20

6. (a) Evaluate the complex integral :

$$\int_C \frac{dz}{\cos h\,(z)},\text{ where C is the }|z|=2.$$

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(b) Prove that :

$$\int_0^\infty \frac{\cos ax}{(x^2+b^2)^2}\,dx = \frac{\pi(ab+1)\,e^{-ab}}{4b^3}$$

(Here $a > 0$, $b > 0$) using Cauchy Residue theorem.

(c) Prove that, by contour integration the real integral :

$$\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\,d\theta = \frac{\pi}{12}$$ 20, 20, 20

7. (a) Form the partial differential equation from :

(i) $z = x\,f(ax+by)+g(ax+by)$,

Where a, b are arbitary constants.

(ii) $F(xy+z^2,\ x+y+z)=0$

Where $f(.)$ is an arbitary function.

(b) Apply Charpits method to find the complete integral of :

$$z^2(z^2p^2+q^2)=1$$

(c) Solve the partial differential equation :

$$(x^2-yz)\,p+(y^2-zx)\,q=z^2-xy$$ 20, 20, 20

8. (a) Solve :

(i) $(D^3-7DD'^2-6D'^3)z=x^2+xy^2+y^3+\cos(x-y)$

(ii) $(D^2+DD'-6D'^2)\,z=y\cos x$

(b) Solve :

$$(D^2-2DD')\;z=e^{2x}+x^3y$$

(c) Solve :

$$\frac{\partial^2 z}{\partial x^2}=a^2z \text{ if } \frac{\partial z}{\partial x}=a\sin y \text{ and } \frac{\partial z}{\partial y}=0 \text{ when } x=0.$$

20, 20, 20

SECTION – B

9. (a) The flate surface of a hemisphere of radius r is cemented to one flate surface of a cylinder of the same radius and of the some material. If the length of the cylinder be l and the total mass M, show that the moment of Inertia of the combination about the axis of the cylinder is given by :

$$Mr^2\left(\frac{l}{2}+\frac{4}{15}r\right)\Big/\left(l+\frac{2r}{3}\right)$$

(b) Define moment of Inertia of a body and discuss its physical significance, derive an expression for the moment of intertia of sphere about an axis tangent to its surface. 30, 30

10. (a) A flywheel in the form of a solid circular disc of mass 500 kg and radius/meter is rotating, making 120 rev/min. Compute the kinetic energy and the angular speed if the wheel is brought to rest in 2 seconds; fraction is to be neglected.

(b) Calculate the velocity of efflux of kerosene oil from a tank in which the pressure is 50 lb wt per square inch above the atmospheric pressure. The density of kerosene is 48lb per C. 30, 30

11. (a) Two equal drops of water are falling through air with a steady terminal velocity of 5 cm/sec. If the drops coalesce, what will be the new terminal velocity.

(b) Discuss of effect of :

(i) Temperature

(ii) Pressure on the viscosity of fluids. 30, 30

12. (a) (i) Find a positive root of $x^4 - x = 10$ using Newton-Raphson's method.

(ii) Determine the root of $x e^x - 2 = 0$ by method of False position.

(iii) Find the Newton's general interpolation polynomial and hence find $f(-3)$ and $f(9)$.

x :	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

(b) Find $y'(50)$ of the tabulated function $y = \log_{10} x$, given below :

x :	50	55	60	65
y :	1.6990	1.7404	1.7782	1.8129

(Forward differences : 414, –36, 5, $h = 5$) 30, 30

13. (a) (i) State three point Gaussian Quadrature formula. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Gauss quadrature formula for $n = 3$.

(ii) Using Trapezoidal rule evaluate $\int_0^1 \frac{\sin x}{x} dx$ by dividing the range into 4 equal parts.

(b) Solve the initial value problem by modified Euler's method,

$$\frac{dy}{dx} = x + y,\ y(0) = 1,\ h = 0.1,$$

$$x = 0, (0.1), 0.3$$ 30, 30

14. (a) Using Runge-Kutta method, find the solution of :

$$\frac{dy}{dx} = 0.25\, y^2 + x^2$$

with initial condition $y(0) = -1$ on $[0, 0.5]$ with $h = 0.1$

(b) Solve the differential equation :

$$\frac{dy}{dx} = x^2 - y,\ y(0) = 1$$

by Picard's method of successive approximations to get the value of y at $x = 1$. Using terms through x^5, compare it with the exact analytical solution. 30, 30

15. (a) Solve the following linear programming problem by Simplex method as given below :

Max. $z = 2x_1 + x_3$,

Such that $x_1 + 2x_2 \geq 4$

$x_1 + x_3 \leq 3$

$x_1, x_2, x_3 \geq 0$

(b) A company produces both interior and exterior paints from two raw materials M_1 and M_2. The following table provides the basic data of the problem :

Materials	Tons of raw material/ton		Maximum daily availability (tons)
	Exterior Paint	Interior Paint	
Raw Material M_1	6	4	24
Raw Material M_2	1	2	6
Profit/ton ($ 1000)	5	4	–

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. The maximum daily demand of interior paint is 2 tons. Formulate it as linear programming problem and find the solution of the primal problem by solving its dual, to determine the best product mix of interior and exterior paints that maximizes the total daily profit.

(c) Minimise $z = x - 5y + 20$

Subject to constraints $x - y \geq 0$

$$-x + 2y \geq 0$$

$$x \geq 3$$

$$y \leq 4$$

$$x, y \geq 0$$

Using penalties (or Big - μ) method. 20, 20, 20

16. (a) (i) Derive the probability function of the poisson distribution.

(ii) An insurance company has discovered that only about 0.1 percent of the population is involved in a certain type of accident each year. If its 10,000 policy holders were randomly selected from the population, what is the probability that not more than 6 of its clients are involved in such an accident next year.

(b) Let X be a continuous random variable with probability density function given by :

$$f(x) = \frac{1}{2}e^{-|x|}, \ -\infty < x < \infty$$

Find the moment generating function (mfg) $M_x t$. Also find $E(X)$ and $V(X)$ by using mgf.

(c) Define the following terms and given one example of each them :

(i) Discrete random variable

(ii) Continuous random variable

(iii) Cumulative distribution function

Prove that $f(x)$ is probability distribution function. Also find

$$P\left(x \leq \frac{1}{2}\right) \text{and } P\left(x \leq \frac{1}{2} | \frac{1}{3} \leq x \leq \frac{2}{3}\right) .20, 20, 20$$