10. (a) A particle moves in a straight line with constant acceleration. During the first second of observation it moves 17m and during the next two seconds it moves 52m. Find its acceleration and initial velocity. 20

(b) A particle moving in a straight line with S.H.M. has velocities u and v, when its distances from the centre are 'a' and 'b' respectively. Prove that the period of motion is $2\pi\sqrt{\frac{a^2-b^2}{v^2-u^2}}$. 20

(c) A body is projected upwards with a velocity of 60 ms^{-1} at an angle of 30° with the horizontal. Find (i) time of flight (ii) horizontal range and (iii) maximum height attained by the body. 20

11. (a) Three forces acting on a particle are in equilibrium. The angle between Ist and IInd is 105° and that between IInd and IIIrd is 120°. Find the ratio of forces. 30

(b) A body of weight 60 kg rests on a rough horizontal plane whose coefficient of friction is 1/4, find the least horizontal force required to move the body. 30

12. (a) Define Christoffel symbols of first and second kind and show that, if $g_{ij}=0$ for $i \neq j$ then $\left\{ \begin{matrix} k \\ ij \end{matrix} \right\} = 0$, whenever $i \neq j \neq k$. Here g_{ij} are components of covarient tensor. 30

(b) Define Frenet's Frame. If c: $(a,b) \rightarrow \mathbb{R}^3$ is a strongly regular curve and $\{T(t), N(t), B(t)\}$ is Frenet's Frame at point $t \in (a,b)$ then show that $B=\frac{C' \times C''}{\|C' \times C''\|}$ and $N=B \times T$. 30

Roll No. .... Total No. of Pages : 4

1(CCEM)0

Mathematics

(15)

Paper—I

Time : Three Hours] [Maximum Marks : 300

Note :— (i) Answers must be written in English.

(ii) Number of marks carried by each question are indicated at the end of the question.

(iii) Part/Parts of the same question must be answered together and should not be interposed between answers to other questions.

(iv) The answer to each question or Part thereof should begin on a fresh page.

(v) Your answers should be precise and coherent.

(vi) Attempt any **five** questions. Question No. **1** is compulsory.

(vii) If you encounter any typographical error, please read it as it appears in the text-book.

1. (a) Let $S=\{t^3+t^2-2t+1, t^2+1, t^3-2t, 2t^3+3t^2-4t+3\}$ and let $W=L(S)$ be the linear span of S. Find basis and dimension of W. Also show that linear span of any set is a vector space. 30

(b) If W_1 and W_2 are two subspaces of a vector space V then show that $\dim.(W_1+W_2)=\dim. W_1+\dim. W_2-\dim.(W_1 \cap W_2)$ 30

2. (a) Find basis for the null space of the following matrix 'A'. 30

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 3 & 2 & 0 \\ 3 & 4 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{bmatrix}$$

(b) Reduce the quadratic form : 30

$4x^2+y^2-8z^2+4xy-4xz+8yz$ to the diagonal form by an orthogonal transformation of co-ordinates.

3. (a) Prove that : 30

$$\log(x+h)=\log x+\frac{h}{x}-\frac{h^2}{2x^2}+.....(-1)^{n-1}\frac{h^n}{n(x+\theta h)^n}$$

(b) Using Taylor's Theorem find approximate value of $\log\left[(1.03)^{1/3}+(0.98)^{1/4}-1\right]$ 30

4. (a) If $u=(x^2+y^2+z^2)^{-1/2}$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$ 30

(b) If $z=\tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that :

$$x.\left(\frac{\partial z}{\partial x}\right)+y.\left(\frac{\partial z}{\partial y}\right)=\frac{1}{4}\sin 2z$$ 30

5. (a) Discuss the continuity of the function :

$$f(x)=\begin{cases}(x-a)\sin\frac{1}{(x-a)} & ;x\neq a\\ k\in\mathbb{R} & ;x=a\end{cases}$$ 20

(b) Show that $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$ does not exist. 20

(c) Discuss the applicability of L.M.V. Theorem to the function $f(x)=x(x-1)(x-2)$ in $[0,1/2]$ 20

6. (a) Find the particular Integral of $(D^2+9).\ y=x\cos x$ 30

(b) Solve the differential equation :

$$xdy-ydx-\sqrt{x^2-y^2}dx=0$$ 30

7. (a) If $y=\sin^{-1}x$ prove that

$$(1-x^2).y_{n+2}-(2x+1)\ y_{n+1}\ x-x^2y_n=0$$ 20

(b) If $y=\left[x+\sqrt{x^2+1}\right]^n$, prove that

$$(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-n^2y=0$$ 20

(c) Solve the Diff. Eq. $\sin y\frac{dy}{dx}=\cos y(1-x\cos y)$ 20

8. (a) Find asymptotes to the curve

$y^3+x^2y+2xy^2+y+1=0$ 20

(b) Find the curvature of the curve $x=4\cos t$ & $y=3\sin t$. Also find the points on the curve where it has maximum value. 20

(c) Find the area enclosed by the parabola $y=x^2$ and line $y=x+2$. 20

9. (a) Prove that a necessary and sufficient condition for a vector function $\overrightarrow{f(t)}$ to have constant direction is $\overrightarrow{f(t)}\times\frac{d\overrightarrow{f(t)}}{dt}=0$ 20

(b) If $\bar{f}(x,y,z)=y\hat{i}+(x-2xz)\hat{j}-xy\hat{k}$,

Evaluate $\int(\nabla\times\vec{f})\cdot\hat{n}\,dS$, where S is the portion of sphere $x^2+y^2+z^2=1$ above xy-plane. 20

(c) Prove that div. $(f\nabla g)-$ div.$(g\nabla f)=f\nabla^2 g-g\nabla^2 f$. 20