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Test Booklet Series

Serial No. 137041

A

SCREENING TEST – 2009

SUBJECT : MATHEMATICS

Time Allowed : ***Two Hours***

Maximum Marks : ***120***

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.

2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C** OR **D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE RESPONSE SHEET.

3. You have to enter your Roll Number on this Test Booklet in the Box provided alongside. *DO NOT* write *anything* else on the Test Booklet.

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4. This Booklet contains **120** items (questions). Each item comprises four response (answers). You will select one response which you want to mark on the Respons Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.

5. In case you find any discrepency, in this test booklet in any question(s) or the Responses, a written representation explaining the details of such alleged discrepency, be submitted within three days, indicating the Question No(s) and the Test Booklet Series, in which the discrepency is alleged. Representation not received within time shall not be entertained at all.

6. You have to mark all your responses ***ONLY*** on the separate Response Sheet provided. See directions in the Response Sheet.

7. All items carry equal marks. Attempt ***ALL*** items. Your total marks will depend only on the number of correct responses marked by you in the Response Sheet.

8. Before you proceed to mark in the Response Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Response Sheet as per instructions sent to you with your Admit Card and Instructions.

9. While writing Centre, Subject, and Roll No. on the top of the Response Sheet in appropriate boxes use **"ONLY BALL POINT PEN"**.

10. After you have completed filling in all your response on the Response Sheet and the examination has concluded, you should hand over to the Invigilator only the Response Sheet. You are permitted to take away with you the Test Booklet.

SEAL

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

(For Rough Work)

Screening Test–2009

SUBJECT : MATHEMATICS

Time Allowed : Two Hours] *[Max. Marks : 120*

1. $(2+5^{1/3})^{1/2}$ is :

(a) an integer

(b) a rational number of the form $\frac{p}{q}$, p and q integers in the lowest form, $q \neq 0, q \neq \pm 1$

(c) an algebraic number

(d) a transcendental number

2. The infinum of the set $\left\{ n^{(-1)^n} : n \in \mathbb{N} \right\}$ is :

(a) -1

(b) 0

(c) $-\frac{1}{2}$

(d) not defined in the set of real numbers

3. For any real number a, $\lim_{n \to \infty} \frac{a^n}{\lfloor n}$ equals :

(a) a

(b) $-a$

(c) infinity

(d) 0

4. The sequence $\left\langle \cos\left(\frac{n\pi}{3}\right) \right\rangle$ is :

(a) unbounded

(b) convergent and hence bounded

(c) divergent

(d) oscillatory

5. The value of b for which $1+e^b+e^{2b}+e^{3b}+\ldots\infty = 9$ is :

(a) $3 \log 2 - 2 \log 3$

(b) $2 \log 3 - 3 \log 2$

(c) $3 \log 2 - \log 3$

(d) $\log 3 - 3 \log 2$

6. The norm of the partition $\{-2, -1.6, -0.5, 0, 0.8, 1\}$ of the interval $[-2, 1]$ is :

(a) 0.8

(b) 0.5

(c) 0.2

(d) 1.1

7. The average value of $\cos x$ on $\left[0, \frac{\pi}{2}\right]$ is :

(a) $\pi/2$

(b) $\pi/4$

(c) $2/\pi$

(d) $4/\pi$

8. Let $\langle b_n \rangle$ be a sequence with nth term as $b_n = n(1+(-1)^n)$ and $A = \limsup b_n$, $B = \liminf b_n$, then :

(a) $A = \infty, B = 0$

(b) $A = 0, B = \infty$

(c) $A = B = 0$

(d) $A = B = \infty$

9. $\lim_{n \to \infty} \frac{1}{n}\left[1 + 2^{1/2} + 3^{1/3} + \ldots\ldots + n^{1/n}\right]$ equals :

(a) 0 (b) 1/2

(c) 1 (d) 2

10. Let f, g be two R-integrable functions on $[a, b]$. Then which of the following is ***not*** necessarily R-integrable on $[a, b]$?

(a) fg

(b) fog, when fog is defined

(c) max $\{f, g\}$

(d) min $\{f, g\}$

11. The series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^P}$ converges

(a) for all real values of P

(b) iff $P > 1$

(c) iff $P < 1$

(d) iff $P = 1$

12. Which of the following functions is discontinuous ?

(a) $\left.\begin{array}{ll} f(x) = x^2 \sin\frac{1}{x}, & x \neq 0 \\ \quad = 0, & x = 0 \end{array}\right\}$

(b) $f(x) = |x| \ \forall x \in \mathbb{R}$

(c) $f(x) = \sqrt{x} \ \forall x \in [0, 4]$

(d) $\left.\begin{array}{ll} f(x) = \frac{1}{x}\sin\left(\frac{1}{x^2}\right), & x \neq 0 \\ \quad = 0, & x = 0 \end{array}\right\}$

13. The function f defined on $[-8\pi, 8\pi]$ as $f(x) = \sin(e^x) - e^{4x}\cos 2x$ is :

(a) uniformly continuous

(b) continuous but not uniformly continuous

(c) discontinuous at finitely many points

(d) discontinuous at uncountably many points

14. Let [.] denote the greatest integer function. Then $I = \int_0^2 [x^2]\, dx$ equals :

(a) 0

(b) $9 - \sqrt{2} - \sqrt{3}$

(c) $7 - \sqrt{2} - \sqrt{3}$

(d) $5 - \sqrt{2} - \sqrt{3}$

15. Let f, g, h, k be four functions defined as :

$f(x) = \sqrt{x}$ on $[0, 2]$

$g(x) = x^2$ on $[0, \infty)$

$h(x) = x^3$ on $[-1, 1]$

$k(x) = \frac{1}{x}$ on $]0, 1]$

Which of the functions defined above are ***not*** uniformly continuous in the specified domains ?

(a) g and k

(b) g only

(c) k only

(d) f and h

16. Let f be defined on $\mathbb{R}_+$ as

$f(x) = x$ if $0 \leq x < 1$

$= 1$ if $x \geq 1$

Which one is ***true*** ?

(a) f is discontinuous at $x = 1$

(b) f is differentiable at $x = 1$

(c) f is continuous but not differentiable at $x = 1$

(d) f is a continuous non-monotonic function

17. The solution of the total differential equation :

$(y+z)\,dx + (z+x)\,dy + (x+y)\,dz = 0$ is :

(a) $xy + yz + xz =$ constt.

(b) $x^2 + y^2 + z^2 =$ constt.

(c) $x^2(y+z) + y^2(x+z) =$ constt.

(d) $x + y + z =$ constt.

18. The general solution of the partial differential equation $x^2\frac{\partial v}{\partial x} + y^2\frac{\partial v}{\partial y} = x+y$, where $v = v(x, y)$ is given by :

(a) $\phi\left(y+x, \frac{e^v}{x+y}\right) = 0$

(b) $\phi\left(\frac{1}{x}+\frac{1}{y}, (x+y)\,e^v\right) = 0$

(c) $\phi\left(y-x, \frac{e^v}{x-y}\right) = 0$

(d) $\phi\left(\frac{1}{y}-\frac{1}{x}, \frac{x-y}{e^v}\right) = 0$

19. The complete integral of the differential equation $pq = xz$, (where $p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y},\ z = z(x, y)$) is :

(a) $z = \left(\frac{x^2}{2} + a\right)(y + b)$

(b) $z = (x + a)(y + b)$

(c) $z = ax + a^2 + be^y$

(d) $z = \left(\sqrt{x} + a\right)^2 + \left(\sqrt{y} + b\right)^2$

20. The differential equation of all parabolas, whose axes are parallel to the y-axis is :

(a) $\frac{d^3y}{dx^3} = 0$

(b) $\frac{dy}{dx} - \frac{y}{2x} = 0$

(c) $\frac{d^2y}{dx^2} + 4a\frac{dy}{dx} = 0$

(d) $\frac{d^3y}{dx^3} + 4ax = 0$

21. The solution of the initial value problem $y'' - 3y' + 2y = 0$, $y(0) = 0$, $y'(0) = 1$ is :

(a) $-e^x + e^{2x}$

(b) $e^x - e^{2x}$

(c) $e^x + e^{2x}$

(d) $\frac{1}{2}\left(e^{2x} - e^x\right)$

22. Let A be the set of all sequences with terms 0 or 1. Then the set A is :

(a) finite

(b) countably infinite

(c) uncountable

(d) none of the above

23. Which of the following is true about the set $\mathbb{Z}$ of integers as a subset of real numbers :

(a) $\mathbb{Z}$ is closed

(b) $\mathbb{Z}$ is perfect

(c) $\mathbb{Z}$ is bounded

(d) $\mathbb{Z}$ is compact

24. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^{n^2}}{n}$ is :

(a) 0

(b) 1/2

(c) 1

(d) infinity

25. The region represented by the set $\left\{z : \text{Re}\,(z^2) \leq 1\right\}$ in the complex plane is

(a) a horizontal infinite strip of width 2π

(b) a closed disc without its centre

(c) region between the branches of the hyperbola $x^2 - y^2 = 1$

(d) region enclosed by the ellipse $\frac{x^2}{2} + y^2 = 1$

26. The value of $\int_C \frac{\sin z}{z^2(z-1)}\,dz$, where $C = \left\{z : |z| = \frac{1}{2}\right\}$ is :

(a) $-2\pi i$

(b) $2\pi i$

(c) $4\pi i$

(d) $-4\pi i$

27. The complex function $f(z) = |z|^2$ is :

(a) differentiable only at z = 0

(b) differentiable everywhere in the complex plane

(c) differentiable nowhere in the complex plane

(d) differentiable everywhere in the complex plane except at z = 0

28. The most general harmonic conjugate of the function $u = x^2 - y^2 - y$ is given by

(a) $v(x, y) = x^2 + y^2 - x + c$

(b) $v(x, y) = 2xy + y + c$

(c) $v(x, y) = 2x^2y + y + c$

(d) $v(x, y) = 2xy + x + c$

29. Let $I = \oint_C \bar{z}\,dz$, where C denotes the unit circle traced anticlockwise. Then I equals :

(a) 0

(b) $2\pi i$

(c) πi

(d) $\pi/2$

30. Let $g(z) = (z^2 - 1)^{-1} \tan z$ and C : $|z| = 3/2$ (counterclockwise). Then $I = \oint_C g(z)\,dz$ equals :

(a) 0

(b) $2\pi i \tan 1$

(c) $\pi i \tan 1$

(d) $\pi/2 \tan 1$

31. The solution set of $e^z = -3$ is :

(a) $\{ln\, 3 + (2K + 1)\, \pi i \; : \; K \in z\}$

(b) $\{ln\, 3 + K\, \pi i \; : \; K \in z\}$

(c) $\{ln\, 3 + 2K\pi i \; : \; K \in z\}$

(d) $\{ln\, 3 + \pi i\}$

32. Let f be analytic in a disc D centred at a and having radius R such that $|f(z)| \leq |f(a)|$ for all z in D with $f(a) \neq 0$. Then :

(a) f is a constant function

(b) $\exists$ $z \in D$ such that $f(z) \neq f(a)$

(c) f is identically zero

(d) f is not a rational function

33. The function $f(z) = (1 - \cos z)z^{-2}$ has :

(a) a simple pole at z = 0

(b) a double pole at z = 0

(c) a removable singularity at z = 0

(d) a non-isolated essential singularity at z = 0

34. Let (X, τ) be a topological space of all irrational numbers with discrete topology. Then which of the following is true ?

(a) The space is complete

(b) The space is connected

(c) The space is totally bounded

(d) The space is separable

35. Let f and g be holomorphic inside and on a contour r and $|f(z)| > |g(z)|$ on the boundary of the contour. Then :

(a) g and $f - g$ have the same number of zeros inside r

(b) g and fg have the same number of zeros inside r

(c) f and $f + g$ have the same number of zeros inside r

(d) g and $f + g$ have the same number of zeros inside r

36. The residue of $f(z) = \dfrac{\pi \cot \pi z}{z^2}$ at z = 2 is

(a) 1/2

(b) 1/4

(c) 1/8

(d) 1/16

37. Which one is incorrect :

(a) If H and K are two subgroups of a group G then HK is a subgroup of G iff HK = KH

(b) If H and K are two subgroups of a group G such that either H or K is normal in G then HK is a subgroup of G

(c) If H and K are finite subgroups of a group G, then 0(HK) = 0(H) . 0(K)

(d) If H and K are two subgroups of an abelian group G then HK is a subgroup of G

38. Let $G_1 = \langle a \rangle, G_2 = \langle b \rangle$ be two cyclic groups of orders m and n such that (m, n) > 1. Then which one is not true for the product group G = $G_1 \times G_2$?

(a) G is cyclic

(b) G is abelian but not cyclic

(c) 0(G) = mn

(d) None of the above

39. If in a group G, $a^5 = e$ and $aba^{-1} = b^2$ for a, b $\in$ G, then for b $\neq$ e, 0(b) equals

(a) 29

(b) 31

(c) 33

(d) 35

40. Which one is not true ?

(a) Intersection of two normal subgroups need not be a normal subgroup

(b) Every subgroup of a cyclic group is normal

(c) The centre of a group is a normal subgroup

(d) If $x^2 \in H$ for all $x \in G$, then H is a normal subgroup of G

41. Which one is correct ?

(a) The quaternion group can be written as internal direct product of its non- trivial subgroups

(b) The group Q of all rationals under addition is the direct sum of two non-trivial subgroups

(c) A group of order 4 is cyclic or internal direct product of two cyclic groups of order 2 each

(d) S3, the symmetric group of degree 3, is the internal direct product of two non-trivial subgroups

42. The number of non-isomorphic abelian groups of order 20 is

(a) 1

(b) 2

(c) 4

(d) 5

43. Which of the following is not true for a boolean ring R ?

(a) R is non-commutative

(b) $x^2 = x$ for every x $\in$ R

(c) 2x = 0 for every x $\in$ R

(d) x + y = 0 $\Rightarrow$ x = y for x, y $\in$ R

44. Let R be the ring of real-valued continuous functions on [0, 1] which one of the following is true for the ring R ?

(a) R is an integral domain

(b) R has zero divisors

(c) R is a commutative ring without unity

(d) R is a non-commutative ring with unity

45. If S and T are two subrings of a ring R, then which of the following is always a subring of R ?

(a) S + T

(b) S $\cup$ T

(c) S $\cap$ T

(d) S − T

46. The smallest non-commutative ring is of order :

(a) 2

(b) 3

(c) 4

(d) 5

47. How many idempotents does an integral domain with unity have ?

(a) 0

(b) 1

(c) 2

(d) 4

48. The degrees of $\sqrt{2}+\sqrt{3}$ and $\sqrt{2}+\sqrt[3]{5}$ over Q are respectively :

(a) 4, 6

(b) 2, 6

(c) 6, 2

(d) 6, 4

49. Which one is incorrect ?

(a) A finite normal extension is a minimal splitting field of some polynomial

(b) A normal extension of a normal extension is a normal extension

(c) A quadratic extension is a normal extension

(d) A minimal splitting field of a non-constant polynomial over K is a normal extension of K

50. Which of the following statements is not equivalent to the other three ?

(a) K is algebraically closed

(b) Every algebraic extension of K is K itself

(c) Every polynomial f over K splits in K

(d) Every irreducible polynomial over K has degree at least 2

51. A finite extension E/F is a Galois extension if and only if :

(a) it is normal

(b) it is separable

(c) it is both normal and separable

(d) it is neither normal nor separable

52. Let $f(x)=x^4+x^2+1$. Then degree of the splitting field of f(x) over the field of rationals is :

(a) 1 (b) 2

(c) 3 (d) 4

53. According to Fermat's theorem, for any integer a and prime P,

(a) $a^P \equiv a \pmod{P}$

(b) $a^{2P} \equiv a \pmod{P}$

(c) $a^P \equiv P \pmod{a}$

(d) $a^P \equiv P^2 \pmod{a}$

54. Let $f(x) = 0, -\pi \le x < 0$
$= x, 0 \le x \le \pi$

Then the sum of the Fourier series of f at x = 0 and x = $\pm\pi$ respectively are :

(a) $0, \pi/2$

(b) 0, 0

(c) $\pi/2, \pi/2$

(d) $\pi/2, 0$

55. The Fourier-cosine series of a bounded, piecewise monotonic and integrable function on $[0, \pi]$ is given by $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ where :

(a) $a_n = \frac{1}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$

(b) $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$

(c) $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$

(d) $a_n = \frac{2}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$

56. The number of non-zero nilpotent elements in $\mathbb{Z}_{30}$, the ring of integers modulo 30, is :

(a) 0

(b) 4

(c) 10

(d) 15

57. If A and B are two ideals of a commutative ring R with unity such that $A + B = R$ then :

(a) $A \cup B = A$

(b) $A \cup B = B$

(c) $AB = A \cap B$

(d) $A + B = A \cup B$

58. For disjoint events A, B if $P(A) = 0.5$ and $P(A \cup B) = 0.6$ then $P(B)$ equals :

(a) 0.2

(b) 0.1

(c) 0.3

(d) 0.4

59. One bag contains five white and four black balls. Another bag contains seven white and nine black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. The probability that the transferred ball was white is :

(a) 1/5

(b) 14/153

(c) 68/153

(d) 2/3

60. For $f(x) = cx^2 (1 - x)$, $0 < x < 1$, to be a probability density function, the constant c equals :

(a) 10

(b) 12

(c) 15

(d) 1

61. If X has distribution given by $P[x = 0] = P[x = 2] = q$ and $P[x = 1] = 1 - 2q$ for $0 < q \leq 1/2$, then the value of q for which the variance of X is maximum is

(a) 0

(b) 1/2

(c) 1/4

(d) 1/8

62. Which one is not true ?

(a) In a Boolean ring R, every prime ideal $P \neq R$ is maximal

(b) In a commutative ring with unity, every maximal ideal is prime

(c) In a finite commutative ring, every maximal ideal is prime

(d) A commutative ring with unity in which every ideal is prime is a field

63. The polynomial $f(x) = x^2 - 2x - 15$ is :

(a) reducible but not primitive over $\mathbb{Z}$

(b) primitive but not reducible over $\mathbb{Z}$

(c) both primitive as well as reducible over $\mathbb{Z}$

(d) neither primitive nor reducible over $\mathbb{Z}$

64. Let X be a random variable with mean μ and variance σ^2. Then $E[(X-b)^2]$, as a function of b, is minimized when b equals :

(a) $\mu/2$

(b) σ^2

(c) $\sigma^2/2$

(d) μ

65. A box contains *a* white and *b* black balls; *c* white balls are drawn. The expectation of the number of white balls drawn is :

(a) $ca/a+b$

(b) $ab/a+b$

(c) $cb/a+b$

(d) $abc/a+b+c$

66. The mode of a binomial distribution with n = 8 and $\rho = 1/2$ is :

(a) 4

(b) 4, 5

(c) 5

(d) 9

67. If X is a gamma variate with parameters $\lambda > 0$ and $r > 0$ then $\frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$ equals :

(a) $\frac{1}{r}$ (b) $\frac{\lambda}{r}$

(c) $\frac{1}{\sqrt{r}}$ (d) $\frac{\lambda}{\sqrt{r}}$

68. If $\rho(X, Y)$ denotes the correlation coefficient of two random variables X and Y then :

(a) $\rho(X, Y) \geq 2$

(b) $\rho(X, Y) > 0$

(c) $-1 \leq \rho(X, Y) \leq 1$

(d) $0 \leq \rho(X, Y) \leq 1$

69. The remainder when the sum $1^5 + 2^5 + 3^5 + ... + 100^5$ is divided by 4 is :

(a) 1 (b) 3

(c) 2 (d) 0

70. In the plane $\mathbb{R}^2$, the set $\{(x, y) : x \geq 0$ and $y \geq 0\}$ is :

(a) both open as well as closed

(b) neither open nor closed

(c) open

(d) closed

71. Which one is not true ?

(a) Every order topology is Hausdorff

(b) The product of two Hausdorff spaces is Hausdorff

(c) Every subspace of a Hausdorff space is Hausdorff

(d) X is Hausdorff iff $\Delta = \{x \times x : x \in X\}$ is open in $X \times X$

72. For the normal distribution, the quartile deviation, the mean deviation and standard deviation are approximately :

(a) 10 : 12 : 15

(b) 12 : 10 : 15

(c) 15 : 10 : 12

(d) 12 : 15 : 10

73. If (X, Y) has a bivariate normal distribution, then X and Y are independent iff the correlation coefficient $\rho(X, Y)$ equals :

(a) –1

(b) 0

(c) 1

(d) 1/2

74. If X is a random variable such that $E(x) = 3$ and $E(x^2) = 13$, then (using Chebyshev's inequality), the lower bound for $P(-2 < x < 8)$ is :

(a) 21/25

(b) 19/25

(c) 4/5

(d) 17/5

75. Let $\{X_k\}$ be a sequence of independent variates such that $P[X_k = \pm 2k^\alpha] = 1/2$. The value of α for which weak law of large numbers holds is :

(a) $\alpha > 1/2$

(b) $\alpha = 1/2$

(c) $\alpha = 1$

(d) $\alpha < 1/2$

76. Let $S = \left\{\left(x, \sin\frac{1}{x}\right) : 0 < x \leq 1\right\}$ be a subset of the plane which one is incorrect ?

(a) S is connected

(b) The closure of S, i.e. $\overline{S}$ is connected

(c) The closure of S is path connected

(d) S is a continuous image of a connected set

77. The subspace $[-1, 0) \cup (0, 1]$ of $\mathbb{R}$ is :

(a) connected and hence locally connected

(b) locally connected but not connected

(c) neither connected nor locally connected

(d) connected but not locally connected

78. Let C denote the Cantor set, as a subspace of [0, 1] which of the following is not true about C, :

(a) C is totally disconnected

(b) C is compact

(c) C is uncountable

(d) C has isolated points

79. The one-point compactification of $\mathbb{Z}_+$ is homeomorphic with :

(a) the subspace $\{0\} \cup \left\{\frac{1}{n} / n \in \mathbb{Z}_+\right\}$ of $\mathbb{R}$

(b) the circle S^1

(c) the sphere S^2

(d) the interval (0, 1)

80. According to Urysohn metrization theorem :

(a) Every regular space X with a countable basis is metrizable

(b) Every Hausdorff space X with a countable basis is metrizable

(c) Every locally compact Hausdorff space X is locally metrizable

(d) Every regular Lindelöf space X is metrizable

81. Which of the following spaces is a complete metric space ?

(a) The space Q of rationals with the usual metric

(b) The open interval (–1, 1) in $\mathbb{R}$ with the usual metric

(c) The set of all sequences $(x_1, x_2, ...)$ such that $\sum_{l=1}^{\infty} x_i^2$ converges, in the l^2-metric

(d) The set Q of all rational numbers with metric $\frac{(x-y)}{1+|x-y|}$

82. The topological dimension of any compact subspace X of the real line $\mathbb{R}$ is :

(a) exactly 1

(b) at most 1

(c) at least 1

(d) infinite

83. The torsion of the curve $\vec{r} = \vec{r}(u) = (u, u^2, u^3)$ is given by :

(a) $\frac{3}{9u^4 + 9u^2 + 1}$

(b) $\frac{1}{9u^4 + 9u^2 + 1}$

(c) $\left(\frac{1}{9u^4 + 9u^2 + 1}\right)^{1/2}$

(d) $\left(\frac{2}{9u^4 + 9u^2 + 1}\right)^{1/2}$

84. Suppose that a space curve drawn on the surface of a cylinder has the property that at each point on it the ratio of curvature to torsion is constant, then the curve must be :

(a) a straight line

(b) a plane curve

(c) a helix

(d) a Bertrand curve

85. According to Meunier's theorem, at a point common to the curve and the surface,

(a) $K_n/K = \cos\phi$ where ϕ is the angle between $\vec{n}$ and $\vec{N}$

(b) $K. K_n = \cos\phi$ where ϕ is the angle between $\vec{n}$ and $\vec{N}$

(c) $K + K_n = \cos\phi$ where ϕ is the angle between $\vec{n}$ and $\vec{b}$

(d) $K - K_n = \cos\phi$ where ϕ is the angle between $\vec{t}$ and $\vec{n}$

86. In $\mathbb{R}$ with usual metric, $\bigcap_{n \in \mathbb{N}} \left\{\left(0, 1+\frac{1}{n}\right)\right\}$ is :

(a) an open set

(b) a closed set

(c) both open and closed

(d) not an open set

87. The convex hull of a subset A of $\mathbb{R}^n$ is :

(a) the intersection of all convex sets containing A

(b) the union of all convex sets containing A

(c) the largest convex subset of A

(d) the largest convex superset of A

88. The family of curves $u(x, y) = x^2 - y^2 =$ constant is orthogonal to the family of curves $v(x, y) =$ constant where :

(a) $v(x, y) = xy$

(b) $v(x, y) = x + y$

(c) $v(x, y) = x/y$

(d) $v(x, y) = x - y$

89. Let f and g be two bounded monotonically increasing functions defined on [0, 4]. Then f – g is :

(a) continuous on [0, 4]

(b) continuous except for finitely many removable discontinuties

(c) a function of bounded variations on [0, 4]

(d) a step function

90. Which of the following is not a convex set ?

(a) $\{(x, y) : 2 \le x^2 + y^2 \le 8\}$

(b) $\{(x, y) : x^2 + y^2 \le 4\}$

(c) $\{(x, y) : 2x + 4y \le 8\}$

(d) $\{(x, y) : x > 0\}$

91. $\int_0^3 x\, d([x] - x)$ equals :

(a) 5/2

(b) 3/2

(c) 1/2

(d) 9/2

92. $\int_1^\infty \frac{1}{x} dx$ equals :

(a) 0

(b) 1

(c) 1/2

(d) infinity

93. Let f be defined on [0, 3] as

$$f(x) \begin{array}{l} = 1 \text{ if } x \text{ is rational} \\ = 2 \text{ if } x \text{ is irrational} \end{array}\Bigg\}$$

Then :

(a) f is Riemann integrable on [0, 3]

(b) f is Lebesgue integrable but not Riemann integrable on [0, 3]

(c) f is not Lebesgue integrable on [0, 3]

(d) f is both Riemann integrable as well as Lebesgue integrable on [0, 3]

94. The solution set of the linear congruence $12x \equiv 8$ (mod. 15) is :

(a) $\{3 + 15K : K \in \mathbb{Z}\}$

(b) $\{4 + 15K : K \in \mathbb{Z}\}$

(c) $\{7 + 15K : K \in \mathbb{Z}\}$

(d) Empty

95. Which one is correct ?

(a) The set {1, 2, 3, 6, 10, 12, 24, 60, 120} ordered by divisibility is a modular lattice

(b) The class of all normal subgroups of a group is a modular lattice under set inclusion

(c) A modular lattice contains a pentagonal sublattice

(d) The class of all subgroups of a cyclic group of order 8 is a non-modular lattice under set inclusion

96. Let L and M be two chains each containing at least two elements. Then their cardinal product L × M :

(a) is a chain

(b) is a lattice but not a chain

(c) is not a lattice and hence not a chain

(d) is a chain but not a lattice

97. A chain of three or more elements is :

(a) complemented

(b) relatively complemented

(c) neither complemented nor relatively complemented

(d) both complemented as well as relatively complemented

98. Which of the following linear spaces is non-separable ?

(a) C[a, b], the space of all continuous real-valued functions on [a, b]

(b) l^1

(c) l^2

(d) l^∞

99. Let X be a normed space and a mapping $d^* : X \times X \to \mathbb{R}$ be defined as $d^*(x, y) = \min \{1, \|x - y\|\}$, x, y ∈ X; where $\|.\|$ denotes the norm of X. Then :

(a) d* is not a metric on X

(b) d* is a metric on X and there is no norm on X which generates d*

(c) d* is a metric on X generated by some norm on X

(d) none of the above

100. Which one is true ?

(a) An infinite dimensional subspace of a normed space is always closed

(b) In a finite dimensional normed space, every non-empty set is compact

(c) If $||.||$ and $||.||'$ are equivalent norms on X then $(X, ||.||)$ is a Banach space iff $(X, ||.||')$ is so

(d) An unbounded metric on X can never be equivalent to a bounded metric on X

101. Which of the following is not a self-dual space ?

(a) $\mathbb{R}^n$, with usual norm

(b) l^1

(c) l^2

(d) $L^2[a, b]$

102. Let T be a real, skew symmetric $n \times n$ matrix where n is odd. Then det T, the determinant of T equals :

(a) 0

(b) 1

(c) −1

(d) 1/2

103. Which of the following is not true for F-distribution ?

(a) F-distribution has two parameters ν_1 and ν_2

(b) For large values of ν_1 and ν_2, the distribution approaches normal distribution

(c) The random variate F can take any real value

(d) The curve is positively skewed

104. The number of elements in S_3, the symmetric group of degree 3, satisfying $x^2 = e$ is :

(a) 1

(b) 2

(c) 3

(d) 4

105. Let d_1 and d_2 be two metrics on a non-empty set X and

$$d(x, y) = d_1(x, y) + d_2(x, y)$$

$$d^*(x, y) = \max. \{d_1(x, y), d_2(x, y)\}$$

for all $x, y \in X$. Then which of the following is true ?

(a) d is a metric on X but d* is not a metric on X

(b) d* is a metric on X but d is not a metric on X

(c) d and d* are equivalent metrics on X

(d) d and d* are non-equivalent metrics on X

106. Which of the following is not true ?

(a) The completeness of a metric space depends on its metric structure

(b) A subspace of a complete metric space may not be complete

(c) Completeness is preserved under an isometry

(d) Completeness is preserved under a homeomorphism

107. Which of the following is not a Noetherian ring ?

(a) The ring <Z, +, .> of integers

(b) The ring <Q, +, .> of rationals

(c) The ring Q[x] of polynomials over the rational field Q

(d) The ring of all real valued functions on the real field $\mathbb{R}$

108. How many limit points does the sequence <1, 2, 1, 4, 1, 6, ...> have ?

(a) 1

(b) 2

(c) infinite

(d) none

109. Let $f : [0, 1] \to \mathbb{R}$ be a rational valued continuous function such that $f(1/2) = 1/2$. Then f(0) and f(1) are respectively:

(a) $-1/2, 1$

(b) $1/2, 1/2$

(c) $0, 1$

(d) $-1/2, -1/2$

110. A bounded linear operator on a Hilbert space commuting with its adjoint is called a :

(a) normal operator

(b) normaloid operator

(c) hyponormal operator

(d) quasinormal operator

111. Let $T : l^2 \to l^2$ be defined as : $T(x_0, x_1, \ldots x_k, \ldots\ldots) = (0, x_0, x_1/2, x_2/3, \ldots\ldots)$ Then the spectral radius r(T), of T, equals :

(a) 0

(b) 1/2

(c) 1

(d) $\sqrt{5}$

112. If r(T) and $||T||$ denote the spectral radius and the norm (respectively) of a normal operator T, then :

(a) $r(T) = ||T||$

(b) $r(T) < ||T||$

(c) $r(T) > ||T|| + 1$

(d) $r(T) = 1 + ||T||$

113. Let f and g be measurable real-valued functions on X, F be a real continuous function on $\mathbb{R}^2$. If h is defined on X as $h(x) = F(f(x), g(x))$ for every x in X, then :

(a) h is measurable

(b) h is measurable if F is uniformly continuous

(c) h is non-measurable

(d) h is measurable iff F is a constant function

114. If $p > 0$ and α is real, then $\lim\limits_{n \to \infty} \frac{n^{\alpha}}{(1+p)^n}$ equals :

(a) 1

(b) α

(c) 0

(d) infinity

115. Any non-empty perfect set in $\mathbb{R}^K$ ($K \geq 1$) is :

(a) at most finite, containing at least two elements

(b) a singleton

(c) countable

(d) uncountable

116. The set of limits of subsequences of any sequence in a metric space X form

(a) an open subset of X

(b) a closed subset of X

(c) an unbounded subset of X

(d) a compact subset of X

117. Which of the following entire functions is of order zero ?

(a) a polynomial

(b) e^{az}, $a \neq 0$

(c) cos z

(d) sin z

118. If f(z) is an entire function of order ρ and convergence exponent σ, then :

(a) $\sigma > \rho$

(b) $\sigma = \rho$

(c) $\sigma \leq \rho$

(d) σ and ρ not order-related

119. If g is a polynomial of degree d, then the order of $e^{g(z)}$ is :

(a) 1

(b) d

(c) d^2

(d) 0

120. For the function $f(z) = e^z$, $z = \infty$ is :

(a) an isolated essential singularity

(b) a removable singularity

(c) a non-isolated essential singularity

(d) a pole