Roll No. ..... **Total No. of Pages : 11**

1(CCE.M)3

Statistics–II

(23)

Time : Three Hours] [Maximum Marks : 300

INSTRUCTIONS

(i) Answers must be written in English.

(ii) The number of marks carried by each question is indicated at the end of the question.

(iii) The answer to each question or part thereof should begin on a fresh page.

(iv) Your answer should be precise and coherent.

(v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.

(vi) Candidates should select any **three** Sections. Attempt any **five** questions from the selected Sections, choosing at most **two** questions from selected Sections.

(vii) If you encounter any typographical error, please read it as it appears in the text-book.

(viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.

(ix) No continuation sheets shall be provided to any candidate under any circumstances.

(x) Candidates shall put a cross (x) on blank pages of Answer Script.

(xi) No blank page be left in between answers to various questions.

(xii) No programmable Calculator is allowed.

(xiii) No stencil (with different markings) is allowed.

SECTION–I

1. (a) Discuss the basic principles of a Sample Survey.

(b) Show that if a random sample of size n is drawn without replacement from a finite population of size N with mean μ and variance σ^2, the covariance between any two members of the sample is : $-\sigma^2/N-1$. 20

(c) Describe the advantages of stratified random sampling with illustrations. What are the various methods of allocating a sample in stratified sampling ? 20

(d) Define systematic sampling. Obtain the sampling variance of mean based on simple random sampling and compare the variance with that based on simple random sampling and stratified random sampling. 20

2. (a) Explain the principles of replication, randomisation and local control in experimental designs. Explain how these principles are used in Randomised Block Design and Latin Square Design. 20

(b) How is efficiency of a design measured ? Derive the expressions to measure the efficiency of Latin Square Design over a Randomised Block Design when rows are used as blocks. 20

(c) What do you understand by a T-scale ? Explain clearly the method of converting raw test scores into T-scores. Show that this scaling procedure helps in the process of normalizing a skew distribution. 20

SECTION–IV

13. (a) What is time series ? Describe the models of a time series. Which of the models is considered to be more useful and why ? 20

(b) Describe the method of least squares. Explain how you proceed to fit an exponential trend of the type $y = ab^x$. 20

(c) Distinguish between the seasonal and cyclical variations. Discuss the different methods of determining seasonal variations in a time series. 20

14. (a) What is meant by a demand function ? Explain price and income elasticity of demand. 15

(b) Describe the terms heteroscedasticity and multi-collinearity. 10

(c) Define various Weighted Index Numbers and prove that the Marshall-Edgeworth number lies between Laspeyre's and Paasche's index numbers. 20

(d) What is meant by Cost of Living Index Number ? What are its uses and limitations ? 15

15. (a) Describe the nature of the component of a time series. Explain the additive and multiplicative models of a time series stating clearly the assumptions and discuss their relative merits. 20

(b) Describe the different methods for determining trend in a time series. Explain any one of the methods of fitting trend to time series. 20

(c) Outline the mathematical tests for an ideal index number. Illustrate these with respect to Fisher's ideal index number. 10

(d) Let Y be the life length of a Series system of n independent components. Let Y_i, the life length of component i, has exponential distribution with mean i/λ_i, i = 1, 2, ..., n. Then show that Y has exponential distribution with mean $1/\lambda$, where $\lambda = \sum_{i=1}^{n} \lambda_i$. 15

7. (a) What do you understand by Statistical Quality Control (SQC) ? Discuss briefly its need and utility in industry. Examine the need for quality control techniques in production. 20

(b) Explain what is single sampling plan and double sampling plan. Discuss the relative merits and demerits of single and double sampling plans. 20

(c) Under what circumstances would you advocate the use of an Average Outgoing Quality Limit (AOQL) sampling plan ? Derive the Average Outgoing Quality (AOQ) function for a single sampling attribute plan. 20

8. (a) What do you mean by a bridge system structure ? Explain the concept through a diagram and find out the probability of failure for such a system. 20

(b) What do you mean by censoring mechanism in reliability ? Distinguish between Type I and Type II censoring mechanism. Find out maximum likelihood estimators for scale parameter of single parameter exponential distribution :

(i) with no censoring

(ii) with type II censoring. 20

(c) Explain the concepts of series-parallel system structure with system level redundancy and with component level redundancy. Find out probability of failure in both cases. 20

SECTION—III

9. (a) Describe different models used in Operations Research, giving suitable examples. 15

(b) Explain the terms :

(i) Markov process

(ii) Transition probabilities

(iii) Ergodic process. 15

(c) Discuss M/M/I and M/G/I queuing models. 15

(d) State and prove the Markovian property of inter-arrival times. 15

10. (a) Explain in detail any one method for solving a transportation problem. 15

(b) Explain, how the theory of replacement is used in the following problems :

(i) Replacement of items whose maintenance cost varies with time.

(ii) Replacement of items that fail completely. 15

(c) Describe decision rules for a purchase inventory model with one price break, with two price breaks. 15

(d) Discuss different costs associated with an inventory problem. 15

11. (a) Describe the constituents of a queuing system. For the M/M /I (FIFO/∞) system, obtain the steady state solution and calculate expected system length (L), expected waiting time (W) and show that $L = \lambda W$, where λ is mean arrival rate. 20

(b) What is the significance of control structure in FORTRAN language ? Describe the function of SELECT CASE CONSTRUCT with the help of an example. 20

(c) State the similarities and differences between Transportation problem and Assignment problem. Give a computational procedure for solving the transportation problem. 20

12. (a) What is meant by duality in linear programming ? State the complementary slackness property and use it to solve the following linear programming problem :

Minimize : $z = 2x_1 + 3x_2 + 5x_3 + 2x_4 + 3x_5$

subject to : $x_1 + x_2 + 2x_3 + x_4 + 3x_5 \geq 4$

$2x_1 - 2x_2 + 3x_3 + x_4 + x_5 \geq 3$

and $x_1, x_2, x_3, x_4, x_5 \geq 0$. 20

(b) Describe the constituents of a queuing system. For M/M/S (FIFO/∞) system, obtain steady state solution, mean system length and mean queue length. 20

(c) Define function sub program. What are the basic rules for writing a function sub program ? How a function sub group can be called in the main program ? 20

(c) Define a Balanced Incomplete Block Design (BIBD) and derive the equality and inequality relations among its parameters. Derive a necessary condition for the existence of a symmetric BIBD with even number of treatments. 20

SECTION—II

5. (a) Explain the role of statistical quality control in industry. 15

(b) Derive the control limits for $\bar{x}$ and R-charts when :

(i) Standards are known

(ii) Standards are not known. 15

(c) Explain the procedure of construction of control charts for the number of defects. 15

(d) Describe the method of double sampling plan and derive its O.C., A.O.Q., A.S.N., and ATI. 15

6. (a) Define :

(i) Reliability

(ii) Structure function

(iii) K-out-of-n structure

Write down the structure function of a k-out-of-n structure. 15

(b) Define failure rate, hazard rate and hazard function and obtain these when the random life length of a component has Weibull distribution. 15

(c) Prove or disprove : The distribution of a random life length x has constant failure rate iff it is exponential. 15

(d) Explain :

(i) Time Reversal test

(ii) Factor Reversal test

(iii) Circular test in index number theory. 10

16. (a) Explain clearly with the help of an illustration how seasonal index is useful in planning sales and production for specific periods. Are there any limitations of seasonal index ? 20

(b) Explain the concept of an index number. What are Marshall-Edgeworth, Laspeyre's and Paasche's index numbers ? Prove that the Marshall-Edgeworth index numbers lie between Laspeyre's and Paasche's index numbers. 20

(c) What is the problem of multicollinearity ? Discuss estimation of parameters in the presence of multicollinearity. 20

SECTION—V

17. (a) State the different sources of demographic data. State the limitations and uses of demographic data. 15

(b) Define stable population and stationary population. Show that with usual notations : $m_x = \frac{2q_x}{2 - q_x}$. 15

(c) Explain the structure of a complete life table. How does an abridged life table differ from a complete life table ? 15

(d) Explain the uses of stable and quasi stable population techniques in estimation of demographic parameters. 15

18. (a) Explain the different methods of scaling scores. 20

(b) Explain the importance of reliability and validity in test standardisation. Describe different methods of obtaining reliability coefficient. 20

(c) What do you mean by intelligent quotient ? Describe the test procedure and tests for measuring I. Q. 20

19. (a) What do you mean by the fertility of a population ? Define :

(i) Crude Birth Rate

(ii) General Fertility Rate

(iii) Total Fertility Rate.

Discuss their relative merits and demerits as measures of fertility. 30

(b) Define central death rate, stable population and stationary population. Show that with the usual notations :

(i) $m_x = \frac{2q_x}{2 - q_x}$

(ii) $\overset{o}{e}_x = \frac{T_x}{\ell_x}$

(iii) $L_x = \ell_{x+\frac{1}{2}}$. 30

20. (a) Define 'standardized scores' and 'normalized scores' and describe how they are derived. 20

(b) Discuss the various methods used to determine the reliability of test scores.

In a group of 100 students of B.Sc. class, the reliability coefficient for an achievement test is 0.75. What will be the effect upon reliability of test, if the length of the test is tripled ? 20

(c) Describe the factorial method of experimentation. Give the expression for the total effect, the main effect and S.S. due to an effect and the Standard error of an effect for a 2^n-experiment. 20

3. (a) Define simple random sampling with replacement and simple random sampling without replacement scheme of sampling. Find the expressions for the variance of sample mean in both the cases. 20

(b) Define ratio and product estimator for population mean and also find the variance of these estimators, up to terms of $O(n^{-1})$. Find the conditions under which these estimators are more efficient than sample mean. 20

(c) Define two stage sampling. If n denotes the number of First Stage Units (fsu) in the sample and m denotes the Second Stage Units (ssu) to be drawn from each selected fsu. Find the optimum value of m and n for a fixed cost C given by $C = c_1 n + c_2 nm$. 20

4. (a) Obtain an unbiased estimator of the population total based on a sample drawn with probability proportional to size (pps) with replacement and find its variance. Also find gain due to pps sampling over simple random sampling with replacement. 20

(b) Distinguish between partial and total confounding in factorial experiments. Derive the full details of the analysis of a 2^3 randomized blocks factorial experiment involving $2l$ blocks, each block consisting of 4 plots, in which none of the main effects is confounded in any replicate and no effect is totally confounded. 20