(b) A TV repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately Poisson with an average of 10 per 8 hour day, what is the repairman's expected idle time each day ? How many jobs are ahead of the average set just brought in ? 20

(c) Determine the optimal sequence of jobs which minimizes the total elapsed time based on the following information.

Processing time on the machines A, B, C :

Job	A_i	B_i	C_i
1	3	3	5
2	8	4	8
3	7	2	10
4	5	1	7
5	2	5	6

20

Roll No. .... **Total No. of Printed Pages : 10**

1(CCE.M)2

Mathematics—II

(15)

Time : Three Hours] [Maximum Marks : 300

INSTRUCTIONS

(i) Answers must be written in English.

(ii) The number of marks carried by each question is indicated at the end of the question.

(iii) The answer to each question or part thereof should begin on a fresh page.

(iv) Your answer should be precise and coherent.

(v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.

(vi) Candidates should attempt **any five** questions.

(vii) If you encounter any typographical error, please read it as it appears in the text book.

(viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.

(ix) No continuation sheets shall be provided to any candidate under any circumstances.

(x) Candidates shall put a cross (×) on blank pages of Answer Script.

(xi) No blank page be left in between answer to various questions.

(c) The following table gives the wholesale price index (x) and the index of agricultural production (y) for a period of 5 years :

x	100	104	102	103	106
y	111	109	113	107	110

Draw the scatter diagram and find the coefficient of correlation between x and y. 20

14. (a) Let X be a binomial random variable with parameters n and p. Then prove that

$$E(X) = np$$

and $\text{var}(X) = npq,\ p + q = 1$ 20

(b) (i) Three coins are tossed. Find the probability distribution of the number of heads.

(ii) Consider the experiment of tossing a coin till a head appears for the first time. Let X be the number of tosses required. Find the distribution of X. 20

(c) Define hypergeometric distribution.

(i) A box contains 10 screws out of which 3 are defective. Two screws are taken at random from the box. Find the distribution of the number of defective screws drawn.

(ii) Four cards are taken from a well shuffled pack of 52 cards. What is the probability that (I) 2 are black and 2 are red (II) there is no black card ? 20

Show that (X, d) is a metric space. Find the open sphere with center 2 and radius 1 in the metric space $(\mathbb{R}, d)$ where $\mathbb{R}$ is set of reals. 20

(b) Let $\{x_n\}$ be a Cauchy sequence in a metric space (X, d). Prove that $\{x_n\}$ is convergent if and only if it has a convergent subsequence. 20

(c) Define a compact metric space. Show that the set $\mathbb{R}$ of reals is not compact with usual metric. Is the set [0, 1] compact in $\mathbb{R}$? Justify. 20

4. (a) Let $f = [a, b] \to \mathbb{R}$ be bounded. Prove that f is integrable on [a, b] if and only if given $\in > 0$, there exists a partition P of [a, b] such that $U(f, P) - L(f, P) < \in$. 20

(b) (i) Evaluate $\int_0^a x^2\,dx$ using Riemann sums.

(ii) Define $f = [0, 2] \to \mathbb{R}$ by $f(x) = 1$ for [0, 1] and $f(x) = 2$ for (1, 2). Show that f is integrable in [0, 2] and compute the integral of f. 20

(c) (i) Test for convergence of the series :

$$1+\frac{1}{2}-\frac{1}{3}-\frac{1}{4}+\frac{1}{5}+\frac{1}{6}-\frac{1}{7}-\frac{1}{8}+.......$$

(ii) Show that the series

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+.......$$

does not converge. 20

5. (a) (i) Show that the function :

$f(z) = x^2 - y^2 + i2xy$ is analytic throughout z plane.

(ii) Show that the function $f(z) = \overline{z}$ is nowhere analytic in z plane. 20

(b) Prove that in a metric space every convergent sequence is Cauchy. Is the converse true ? Justify. 20

(c) Consider the set $\mathbb{R}$ of all reals with usual metric and define a function $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2$. Show that f is continuous, but not uniformly continuous. 20

SECTION—B

9. (a) Find a real root of the equation $x^3 - x - 11 = 0$ correct to two decimal places using bisection method. 20

(b) Find a real root of the equation $x^3 - 2x - 5 = 0$ correct to two decimal places using regula falsi method. 20

(c) Prove that the Newton-Raphson method for finding root of the equation $f(x) = 0$ has a quadratic convergence. 20

10. (a) The data below gives the values of tan x for $0.10 \le x \le 0.30$:

x :	0.10	0.15	0.20	0.25	0.30
y = tan x :	0.1003	0.1511	0.2027	0.2553	0.3093

Using Newton's forward difference interpolation formula find tan 0.12. 20

(b) Estimate the value of the integral $\int_1^3 \frac{1}{x}\,dx$ by Simpson's rule with 4 sub intervals. Determine the error by direct integration. 20

(c) Use the Runge-Kutta fourth order method to find y(1) given that $y(0) = 1$ and that $\frac{dy}{dx} = \frac{y-x}{y+x}$. 20

11. (a) Solve the following l.p.p. programming problem by Simplex method :

$$\max.\ z = x_1 + 2x_2 + 3x_3 - x_4$$

such that

$$x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 = 10, \quad x_1, x_2, x_3, x_4 \ge 0.$$ 20

(b) Solve the following transportation problem for minimum cost :

	D_1	D_2	D_3	D_4	a_i
O_1	1	2	−2	3	70
O_2	2	4	0	1	38
O_3	1	2	−2	5	32
b_j	40	28	30	42	140

20

(c) The computer centre has got three expert programmers. The centre needs three application programmes to be developed. The head of the computer centre, after studying carefully the programmes to be developed, estimates the computer time in minutes required by the experts to the application programmes as follows :

		Programme		
		A	B	C
Programmer	1	120	100	80
	2	70	90	110
	3	110	140	120

Assign the programmers to the programmes in such a way that the total computer time is least. 20

(b) State and prove Cauchy's Integral formula. 20

(c) Evaluate the following integrals :

(i) where C is the circle | z | = 1.

(ii) $\int_C \frac{1}{z^2+4} dz$ where C is the circle | z – i | = 2. 20

6. (a) (i) Eliminate the arbitrary function F from the equation z = x + y + F(xy) and find the corresponding p.d.e.

(ii) Eliminate the parameters a and b from the equation $2z = (ax + y)^2 + b$ and find the p.d.e. 20

(b) Show that z = ax +(y/a) + b is a complete integral of pq =1. This problem has no singular integral. Find the particular solution corresponding to the sub-family b = a. 20

(c) Find a complete integral of the p.d.e. $f = (p^2 + q^2)\, y - qz = 0$ by Charpit's method. 20

$\int_C z\, e^{-z}\, dz$

7. (a) Show that the set of all integers is a ring with respect to usual addition and multiplication of integers. Is the ring commutative ? Justify. 20

(b) Define an ideal in a ring. Show that the set $3\mathbb{Z}$ of all multiples of 3 is an ideal of the ring of integers. 20

(c) Prove that a finite integral domain is a field. Give an example of an infinite integral domain that is not a field. 20

8. (a) Let $\mathbb{R}$ be a set of all reals and $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is defined by $d(x, y) = |x - y|$ for $(x, y) \in \mathbb{R} \times \mathbb{R}$. Show that d is a metric on $\mathbb{R}$. Find the open sphere centered at 2 and radius 1 in this metric space. 20

12. (a) Prove that under the three conditions of a Poisson process, the number of units serviced in a fixed time also follows the Poisson Law. 20

(b) If for a period of 2 hours in a day say 8 to 10 a.m., trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period :

(i) the probability that the yard is empty.

(ii) average queue length on the assumption that the line capacity of the yard is limited to 4 trains only. 20

(c) Find the sequence that minimizes the total elapsed time required to complete the following jobs :

	Processing time in hours					
No. of job	1	2	3	4	5	6
Machine A	4	8	3	6	7	5
Machine B	6	3	7	2	8	4

20

13. (a) Explain the method of construction of :

(i) subdivided bar diagram

(ii) percentage bar diagram

(iii) pie chart

Indicate the situations where they are useful. 20

(b) (I) What is the probability of getting :

(i) exactly two heads in tossing 3 coins ?

(ii) at least two heads in tossing 3 coins ?

(II) Find the probability that the sum of the numbers shown in the two faces, when two dice are thrown :

(i) is 7 and

(ii) is 10. 20

(xii) No programmable Calculator is allowed.

(xiii) No stencil (with different markings) is allowed.

SECTION–A

1. (a) Prove that every group is isomorphic to a permutation group. 20

(b) Show that the set of integers $\mathbb{Z}$ is a group with respect to usual addition. Is this group cyclic ? Justify your answer. 20

(c) Express the following permutations as a product of disjoint cycles and determine whether they are even or odd permutations :

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 6 & 4 & 3 \end{pmatrix}$

(ii) $(4\ \ 2\ \ 1\ \ 5)(3\ \ 4\ \ 2\ \ 6)(5\ \ 6\ \ 7\ \ 1)$ 20

2. (a) Suppose I and J are two ideals in a ring R. Prove that $I \cap J$ is an ideal in R. Show by an example that union of two ideals of a ring may not be an ideal of the ring. 20

(b) Define a Unique Factorization Domain (UFD). Show that $\mathbb{Z}$ is α UFD. Give an example of an integral domain that is not UFD. 20

(c) Prove that every field is an integral domain. Is the converse true ? Justify your answer. 20

3. (a) Let X be an arbitrary non empty set and define

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}.$$

15. (a) Solve the following l.p.p. by Simplex method :

Max. $z = 3x_1 + 5x_2 + 4x_3$

such that $2x_1 + 3x_2 \leq 8$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$2x_2 + 5x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$ 20

(b) Solve the following transportation problem :

	D_1	D_2	D_3	a_i
O_1	7	3	3	4
O_2	3	1	4	1
O_3	4	3	6	5
b_j	2	3	5	

20

(c) Solve the following minimal assignment problem :

Job\Man	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

20

16. (a) Prove that if T, the time between two successive arrivals, is a random variable, then it follows an exponential distribution with parameter λ, where the arrival pattern follows a Poisson process. 20